

Bayesian Analysis on Quantitative Decision-Making

Elliot Reedy (MSc)

Student of the PhD in Mathematics at the Department of Science in the School of

Doctoral Studies of the EU

Square de Meeus 37-4th Floor 1000 Brussels, Belgium

[email: elliott.reedy.math@sdcsds.eu](mailto:elliott.reedy.math@sdcsds.eu)

Abstract

Quantitative decision-making is thought of most often as an objective exercise based only on the cold analysis of verifiable hard facts. Intuition and even experience tends to be excluded from quantitative decision-making on the grounds that such information is subjective in character, and, thus, has no role in quantitative analysis.

Quantitative decision-making is based in large part on the ability of decision-makers to make inferences about the probabilities of occurrence of future events from the analyses of objective data (Markowitz and Xu 60-69). One means of improving probability estimates in such predictions, however, is the application of Bayes' Theorem (Peebles 17-19). Classical statistics are concerned with the analysis of sampled data. The analysis of sampled data permits the researcher to make inferences concerning total populations, with the exclusion of any personal judgment or opinions. To the contrary, Bayesian statistics purposefully incorporates informed judgments into the analyses of data. Informed judgments are based on sound experience. While such judgments may be termed intuition, they are not irrational.

This research examines the role of the inclusion of informed judgments based on experience into quantitative decision-making. This approach to quantitative analysis is called Bayesian statistics or Bayesian analysis (Zellner 5).

Key words: Decision Making, Bayesian Analysis, Econometrics.

Bayesian Analysis

Bayesian statistics are based on Bayes' theorem, which is concerned with the derivation of a conditional probability (Summers, Peters, and Armstrong 645). A conditional probability is an alteration of the probability found through the application of classical statistical analysis.

Bayes' theorem was developed by Thomas Bayes, an English clergyman and mathematician of the eighteenth century (Shao 243). In its most simple form, the theorem is a variation on the general formula for conditional probability. The significance of Bayes' theorem is that it is applied in the context of sequential events, and further, that the computational version of the formula provides the basis for determining the conditional probability of an event having occurred in the first sequential position given that a particular event has been observed in the second sequential position (Davis 573-579).

Bayesian statistics has been expanded to incorporate the expected value criterion, which is often referred to as the Bayesian criterion (Pfaffenberger and Patterson 154-159). Use of the Bayesian criterion is a distinct procedure (involving the use of expected values) from that of the Bayesian theorem—involving the revision of prior probability values.

The principal techniques of classical inference are interval estimation and hypothesis testing. By contrast, the primary concern of Bayesian decision analysis is the choice of a decision act (Pfaffenberger and Patterson 154-159). Although classical statistical techniques are directly concerned with estimating values or with the testing of hypotheses concerning population parameters, the results of these procedures are related to alternative courses of action or to alternative decisions. As an example, the acceptance of a null hypothesis (or the inability to reject a null hypothesis) in classical statistical analysis which indicated that the sales level for a product

will be below the breakeven point for that product typically would be associated also with a decision not to market the product. Bayesian statistical analysis also is concerned with the selection of the best decision.

The primary differences between classical statistical analysis and Bayesian statistical analysis are the use of subjective information in the Bayesian analyses, and the evaluation of alternative decision actions within the context of economic consequences (Pfaffenberger and Patterson 154-159). In Bayesian statistical analysis, economic consequences may be formulated as either conditional values (payoffs) or as conditional opportunity losses (regrets).

Essentially, the choice between probability levels of Type I and Type II errors is the basis on which the relative importance of two alternative types of mistakes are assessed in hypothesis testing in classical statistical analysis (Emory 420). The use of the opportunity loss concept in Bayesian statistical analysis represents a similar approach to evaluation, but does so in a more explicit manner. Whereas classical statistical analysis decision procedures are based entirely on the analysis of data collected through a random sampling of the total population, the decision procedures associated with Bayesian statistical analysis may include the analysis of sample data, but are not dependent solely upon the availability of such data.

From the perspective of practicality, an important plus for Bayesian statistical analysis, as opposed to classical statistical analysis, is that decision analysis begins with an identification of managerial judgments, which may then be included in the analysis (Peebles 24). This approach to quantitative analysis means that statistical analysis personnel must work in close coordination with the managerial personnel within an organization—a plus for the procedure in and of itself. By contrast, the exclusive orientation of classical statistical analysis towards sample data does not provide managerial personnel with the opportunity to present their informed judgments for inclusion in statistical decision analyses, or to gain the feeling that informed judgments on their part are viewed as significant by statistical analysis personnel.

One of the principal applications of Bayesian statistical analysis is the derivation of alternative probabilities to those which may be observed in sample data (Summers, Peters, and Armstrong 612). Bayes' theorem is used for the purpose of altering probabilities associated with an entire set of probable events, or states,

in a decision situation. In the context of such a decision situation, prior probability distribution is that probability distribution which is applicable before any sample data are collected. In Bayesian decision analysis, this type of probability distribution is often subjective, in that the data upon which it is based is itself based on the judgments of individuals. In Bayesian statistical analysis, however, the prior probability distribution also may be based on historical data.

The posterior probability distribution is the probability distribution which is applicable subsequent to the observation of sample data, and subsequent to the use of these data to revise the prior probability distribution through application of the Bayes' theorem (Pfaffenberger and Patterson 154-159). In order to apply the Bayes' theorem, the prior probability of an uncertain event and the conditional probability of the sample result must be known. Typically, the conditional probabilities are determined by the application of some standard probability distribution according to the character of the sampling situation (Emory 61). Bayesian formulas for the determination of posterior distribution probabilities differ according to the conditions of the prior distribution probability (Summers, Peters, and Armstrong 649).

Posterior analysis in Bayesian analysis is the process by which the value of sample data is established before the sample data are collected (Pfaffenberger and Patterson 154-159). The basic procedure involved is that of considering all of the possible sample outcomes, determining the estimated value in the decision process of each of the possible sample outcomes, and, finally, determining the expected values of the sample data by weighting each of these different values by the probability that the associated sample outcome will actually occur. It can be seen, thus, that Bayesian statistical analysis is the principal theoretical basis for decision-tree analysis, as well as for most of the other quantitative decision-rule analysis procedures (Emory 84).

The use of Bayesian decision analysis in cases where normal distributions apply requires that only two decision acts be evaluated at one time and that the payoff functions associated with these two alternatives are linear (Pfaffenberger and Patterson 154-159).

The prior probability distribution is descriptive of the uncertainty which is associated with the decision maker's estimate of the probability of occurrence of a random event. It is not the event which follows the probability distribution. Rather, it is the estimate of the

event. Since the estimate of the event is based on an informed judgment, there is no mathematical theorem which would justify the use of the normal distribution in respect to such a judgment in any specific situation. For judgment situations in which an informed decision maker is aware of a number of uncertain factors which could influence the value of the eventual outcome in one direction or the other, however, the use of the normal distribution has been found to be a satisfactory approximation of the uncertainty inherent in the estimate (Pfaffenberger and Patterson 154-159). A normal distribution is defined by identifying the mean and the standard deviation of the distribution. The mean of the prior distribution may be obtained by asking the decision maker to identify the most likely value of the random event or by asking the decision maker for that value which reflects a 50.0 percent chance that the actual value will be lower and a 50.0 percent chance that the actual value will be higher. The first approach is, essentially, a request for the mode of the probability distribution, while the second approach is a request for the median of the distribution. The mean, median, and mode are all at the same point for a normally distributed variable.

The existence of a linear payoff function is an indication that the expected payoff value associated with a specific act is a linear function with respect to the uncertain level of the state (Summers, Peters, and Armstrong 615). The point at which two linear payoff functions cross indicates the point of indifference in respect to making a choice between the two alternative courses of action, because the conditional values of the acts are equal at that point. This point of indifference is called the breakeven point in Bayesian statistical decision-analysis. The specific value of the random variable may be established quantitatively (algebraically) by setting the two equations (for alternative decision choices) equal to one another and solving for the random variable.

Once having established the point of indifference (the breakeven point), the best decision (act) may be determined graphically by observing whether the prior mean is above or below the point of indifference, and by choosing the decision which is optimal on the preferred side of the breakeven point (Pfaffenberger and Patterson 154-159). The best decision (act) also may be identified by substituting the value of the prior mean in each payoff function, and choosing the decision with the highest expected value. It is the latter procedure which is most often employed on Bayesian statistical decision-analysis.

Applying Bayesian Analysis to Decision-Making

The application of Bayesian statistical analysis for purposes of estimation may be best comprehended through the use of example. As the use of Bayesian statistical analysis is most often found in business environment, an example from this environment is used.

Consider that an inventory level determination must be made. A manager knows the probability distribution of a state variable, which, for purposes of this example, is assumed to be product demand in units per day. The inventory problem is to make an optimal point estimate for the demand when there exist certain regrets, or cost consequences, which are associated with either an overestimation or an underestimation of actual demand. The point estimate is used as the order quantity, and, for each unsold unit, there is a cost involved (loss of a perishable product as an example), and, for each unit not available for sale, there is a cost involved (loss of marginal profit as an example).

The total expected regret (cost) is minimized when the incremental expected regret from overestimating equals the incremental expected regret from underestimating. Thus, when the loss per unit of overestimation is equal to the loss per unit of underestimation, the median is the best Bayesian estimate. If the probability distribution of the random variable is symmetrical, the median will coincide with the mean.

A practical approach of Bayesian estimation is a criterion that risk willingness should be proportional. Thus, if the cost per unit of underestimation is four times greater than is the unit cost of overestimation, then a manager should be only one-quarter as willing to risk an underestimate and he or she is willing to risk an overestimation.

Bayesian statistical analysis is an alternative to classical statistical analysis, which provides for the incorporation into the analysis of informed judgments. As such, Bayesian statistical analysis is not suitable for experimental analysis. Bayesian statistical analysis, however, is useful in business and political applications involving actual and opportunity costs where decisions often must be made under uncertain conditions.

Bayesian analysis also has been applied to pricing problems in foreign exchange trading (Flood 952-970). Within this context, the application is through a Bayesian model of market-maker price expectations. For purposes of illustration, consider an expected-profit-maximizing, monopolistic market-maker who faces streams of buy and

sell limit orders from investors. Assume that all orders are for a single round lot; that the market-maker believes that reservation prices of buy orders, are generated by a normal distribution; and that reservation prices of sell orders are generated by a second, independent, normal distribution. Further assume that the market-maker currently holds her or his desired inventory level. The problem in this instance is how should to set her or his spread.

The inventory condition implies that the chosen bid and ask rates must satisfy the constraint so that the expected change in inventory is zero (Flood 952-970). The market-maker is assumed to behave in a Bayesian fashion, using the observed prices on incoming limit orders to refine the parameters of her or his subjective distributions. For example, assume that the market-maker views purchase prices as coming from a normal distribution, but is unsure about the mean of this distribution. Represent this uncertainty by a normal prior density over the possible values for the mean. Given that the marginal subjective density over the prices of incoming limit orders is normal, the market-maker is able to refine her or his subjective distribution of the mean. The posterior analysis permits the market-maker to refine the estimate so that the variance of the marginal subjective price density will be smaller, while the market-maker's optimal spread is narrower.

Conclusion

This research examined the role of intuition and experience in quantitative decision-making. The examination showed that, in the form of informed judgment, experience, or intuition, can play a positive role in quantitative decision-making. The application of informed judgment to the process of quantitative decision-making is accomplished through the performance of Bayesian statistical analysis which is concerned with the derivation of a conditional probability of a future event.

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